```
In[39]:= adc[expr_] := Simplify[
            D[expr, s] * 2 p + D[expr, p] * s * p
          ];
         adc[0, expr_] := expr;
         adc[1, expr_] := adc[expr];
         adc[n_, expr_] /; n > 1 := adc[adc[n - 1, expr]];

In[4]:= adc[1, s]
```

Out[4]= $2\,p$

```
In[5]:= adc[1, p]
```

Out[5]= $p\,s$

```
In[6]:= adc[2, p]
```

Out[6]= $p\,(2\,p + s^2)$

```
In[7]:= adc[3, p]
```

Out[7]= $p\,s\,(8\,p + s^2)$

```
In[8]:= adc[4, p]
```

Out[8]= $p\,(16\,p^2 + 22\,p\,s^2 + s^4)$

```
In[9]:= adc[10, p]
```

Out[9]= $p\,(353\,792\,p^5 + 2\,265\,344\,p^4\,s^2 + 1\,328\,336\,p^3\,s^4 + 136\,384\,p^2\,s^6 + 2026\,p\,s^8 + s^{10})$

```
In[57]:= Table[
          Expand[adc[n, p] /. {s → a + b, p → a b}],
          {n, 0, 9}
         ]
```

Out[57]= $\{a\,b,\ a^2\,b + a\,b^2,\ a^3\,b + 4\,a^2\,b^2 + a\,b^3,\ a^4\,b + 11\,a^3\,b^2 + 11\,a^2\,b^3 + a\,b^4,$
$a^5\,b + 26\,a^4\,b^2 + 66\,a^3\,b^3 + 26\,a^2\,b^4 + a\,b^5,\ a^6\,b + 57\,a^5\,b^2 + 302\,a^4\,b^3 + 302\,a^3\,b^4 + 57\,a^2\,b^5 + a\,b^6,$
$a^7\,b + 120\,a^6\,b^2 + 1191\,a^5\,b^3 + 2416\,a^4\,b^4 + 1191\,a^3\,b^5 + 120\,a^2\,b^6 + a\,b^7,$
$a^8\,b + 247\,a^7\,b^2 + 4293\,a^6\,b^3 + 15\,619\,a^5\,b^4 + 15\,619\,a^4\,b^5 + 4293\,a^3\,b^6 + 247\,a^2\,b^7 + a\,b^8,$
$a^9\,b + 502\,a^8\,b^2 + 14\,608\,a^7\,b^3 + 88\,234\,a^6\,b^4 + 156\,190\,a^5\,b^5 + 88\,234\,a^4\,b^6 +$
$14\,608\,a^3\,b^7 + 502\,a^2\,b^8 + a\,b^9,\ a^{10}\,b + 1013\,a^9\,b^2 + 47\,840\,a^8\,b^3 + 455\,192\,a^7\,b^4 +$
$1\,310\,354\,a^6\,b^5 + 1\,310\,354\,a^5\,b^6 + 455\,192\,a^4\,b^7 + 47\,840\,a^3\,b^8 + 1013\,a^2\,b^9 + a\,b^{10}\}$

```
In[58]:= Table[2^(n + 1) - n - 2, {n, 0, 9}]
```

Out[58]= {0, 1, 4, 11, 26, 57, 120, 247, 502, 1013}

```
In[10]:= Expand[adc[10, p] /. {s → a + b, p → a b}]
```

Out[10]= $a^{11}\,b + 2036\,a^{10}\,b^2 + 152\,637\,a^9\,b^3 + 2\,203\,488\,a^8\,b^4 + 9\,738\,114\,a^7\,b^5 +$
$15\,724\,248\,a^6\,b^6 + 9\,738\,114\,a^5\,b^7 + 2\,203\,488\,a^4\,b^8 + 152\,637\,a^3\,b^9 + 2036\,a^2\,b^{10} + a\,b^{11}$

See http://oeis.org/wiki/Eulerian_numbers,_triangle_of !

In[56]:=
```
Expand[adc[7, p] /. {s → a + b, p → a b}] /. {a → 1, b → 1}
```

Out[56]= 40 320

In[31]:=
```
Expand[CoefficientList[Series[t Exp[x t] / (E^t - 1), {t, 0, 10}], t] *
  Table[k !, {k, 0, 10}]]
```

Out[31]=
$$\left\{1, -\frac{1}{2}+x, \frac{1}{6}-x+x^2, \frac{x}{2}-\frac{3x^2}{2}+x^3, -\frac{1}{30}+x^2-2x^3+x^4, -\frac{x}{6}+\frac{5x^3}{3}-\frac{5x^4}{2}+x^5, \frac{1}{42}-\frac{x^2}{2}+\frac{5x^4}{2}-3x^5+x^6, \frac{x}{6}-\frac{7x^3}{6}+\frac{7x^5}{2}-\frac{7x^6}{2}+x^7, -\frac{1}{30}+\frac{2x^2}{3}-\frac{7x^4}{3}+\frac{14x^6}{3}-4x^7+x^8, -\frac{3x}{10}+2x^3-\frac{21x^5}{5}+6x^7-\frac{9x^8}{2}+x^9, \frac{5}{66}-\frac{3x^2}{2}+5x^4-7x^6+\frac{15x^8}{2}-5x^9+x^{10}\right\}$$

In[24]:=
```
Series[2 Exp[x t] / (E^t + 1), {t, 0, 10}]
```

Out[24]=
$$1+\left(-\frac{1}{2}+x\right)t+\frac{1}{2}\left(-x+x^2\right)t^2+\frac{1}{24}\left(1-6x^2+4x^3\right)t^3+\left(\frac{x}{24}-\frac{x^3}{12}+\frac{x^4}{24}\right)t^4+\left(-\frac{1}{240}+\frac{x^2}{48}-\frac{x^4}{48}+\frac{x^5}{120}\right)t^5+\left(-\frac{x}{240}+\frac{x^3}{144}-\frac{x^5}{240}+\frac{x^6}{720}\right)t^6+\left(\frac{17}{40320}-\frac{x^2}{480}+\frac{x^4}{576}-\frac{x^6}{1440}+\frac{x^7}{5040}\right)t^7+\left(\frac{17x}{40320}-\frac{x^3}{1440}+\frac{x^5}{2880}-\frac{x^7}{10080}+\frac{x^8}{40320}\right)t^8+\left(-\frac{31}{725760}+\frac{17x^2}{80640}-\frac{x^4}{5760}+\frac{x^6}{17280}-\frac{x^8}{80640}+\frac{x^9}{362880}\right)t^9+\left(-\frac{31x}{725760}+\frac{17x^3}{241920}-\frac{x^5}{28800}+\frac{x^7}{120960}-\frac{x^9}{725760}+\frac{x^{10}}{3628800}\right)t^{10}+O[t]^{11}$$

In[25]:=
$$\mathbf{Expand}\left[10!\left(-\frac{31x}{725760}+\frac{17x^3}{241920}-\frac{x^5}{28800}+\frac{x^7}{120960}-\frac{x^9}{725760}+\frac{x^{10}}{3628800}\right)\right]$$

Out[25]= $-155x+255x^3-126x^5+30x^7-5x^9+x^{10}$

In[33]:=
```
Expand[CoefficientList[Series[2 Exp[x t] / (E^t + 1), {t, 0, 10}], t] *
  Table[(k + 1) !, {k, 0, 10}]]
```

Out[33]=
$$\{1, -1+2x, -3x+3x^2, 1-6x^2+4x^3, 5x-10x^3+5x^4, -3+15x^2-15x^4+6x^5, -21x+35x^3-21x^5+7x^6, 17-84x^2+70x^4-28x^6+8x^7, 153x-252x^3+126x^5-36x^7+9x^8, -155+765x^2-630x^4+210x^6-45x^8+10x^9, -1705x+2805x^3-1386x^5+330x^7-55x^9+11x^{10}\}$$

In[11]:=
```
adc[10, s]
```

Out[11]= $2ps\left(176896p^4+230144p^3s^2+40776p^2s^4+1004ps^6+s^8\right)$

In[12]:=
```
Simplify[adc[10, s] /. {s → a + b, p → a b}]
```

Out[12]=
$$2ab(a+b)\left(176896a^4b^4+230144a^3b^3(a+b)^2+40776a^2b^2(a+b)^4+1004ab(a+b)^6+(a+b)^8\right)$$

In[16]:=
```
DSolve[
 {D[s[t], t] == 2 p[t], D[p[t], t] == s[t] p[t]},
 {s[t], p[t]},
 t
]
```

Out[16]=
$$\left\{\left\{s[t] \to \sqrt{2}\sqrt{\left(C[1] + C[1]\left(-1 + \mathrm{Tanh}\left[\frac{1}{2}\left(-\sqrt{2}\, t\sqrt{C[1]} - \sqrt{C[1]}\, C[2]\right)\right]^2\right)\right)},\right.\right.$$
$$\left. p[t] \to \frac{1}{2} C[1]\left(-1 + \mathrm{Tanh}\left[\frac{1}{2}\left(-\sqrt{2}\, t\sqrt{C[1]} - \sqrt{C[1]}\, C[2]\right)\right]^2\right)\right\},$$
$$\left\{s[t] \to -\sqrt{2}\sqrt{\left(C[1] + C[1]\left(-1 + \mathrm{Tanh}\left[\frac{1}{2}\left(\sqrt{2}\, t\sqrt{C[1]} - \sqrt{C[1]}\, C[2]\right)\right]^2\right)\right)},\right.$$
$$\left.\left. p[t] \to \frac{1}{2} C[1]\left(-1 + \mathrm{Tanh}\left[\frac{1}{2}\left(\sqrt{2}\, t\sqrt{C[1]} - \sqrt{C[1]}\, C[2]\right)\right]^2\right)\right\}\right\}$$

In[22]:=
```
DSolve[
 {
  D[g[t, s, p], t] == 2 p D[g[t, s, p], s] + s p D[g[t, s, p], p],
  g[0, s, p] == s
 },
 {g[t, s, p]},
 {t, s, p}
]
```

Out[22]=
$$\mathrm{DSolve}\left[\left\{g^{(1,0,0)}[t, s, p] == p\, s\, g^{(0,0,1)}[t, s, p] + 2\, p\, g^{(0,1,0)}[t, s, p],\ g[0, s, p] == s\right\},\ \{g[t, s, p]\},\ \{t, s, p\}\right]$$

In[44]:=
```
gf = Sum[t^n / n! adc[n, s], {n, 0, 10}] + O[t]^11
```

Out[44]=
$$s + 2\,p\,t + p\,s\,t^2 + \frac{1}{3}\left(2\,p^2 + p\,s^2\right) t^3 + \left(\frac{2\,p^2\,s}{3} + \frac{p\,s^3}{12}\right) t^4 +$$
$$\left(\frac{4\,p^3}{15} + \frac{11\,p^2\,s^2}{30} + \frac{p\,s^4}{60}\right) t^5 + \left(\frac{17\,p^3\,s}{45} + \frac{13\,p^2\,s^3}{90} + \frac{p\,s^5}{360}\right) t^6 +$$
$$\left(\frac{34\,p^4}{315} + \frac{2\,p^3\,s^2}{7} + \frac{19\,p^2\,s^4}{420} + \frac{p\,s^6}{2520}\right) t^7 + \left(\frac{62\,p^4\,s}{315} + \frac{16\,p^3\,s^3}{105} + \frac{p^2\,s^5}{84} + \frac{p\,s^7}{20160}\right) t^8 +$$
$$\left(\frac{124\,p^5}{2835} + \frac{536\,p^4\,s^2}{2835} + \frac{121\,p^3\,s^4}{1890} + \frac{247\,p^2\,s^6}{90720} + \frac{p\,s^8}{181440}\right) t^9 +$$
$$\left(\frac{1382\,p^5\,s}{14175} + \frac{1798\,p^4\,s^3}{14175} + \frac{1699\,p^3\,s^5}{75600} + \frac{251\,p^2\,s^7}{453600} + \frac{p\,s^9}{1814400}\right) t^{10} + O[t]^{11}$$

In[50]:=
```
Simplify[gf * (1 - E^(-s t)) /. {s → a + b, p → a b}]
```

Out[50]=

$$(a+b)^2 t - \frac{1}{2}\left((a-b)^2 (a+b)\right) t^2 + \frac{1}{6}(a+b)^4 t^3 - \frac{1}{24}\left((a+b)\left(a^4 - 18 a^2 b^2 + b^4\right)\right) t^4 + \frac{1}{120}(a+b)^2 \left(a^4 + 4 a^3 b + 46 a^2 b^2 + 4 a b^3 + b^4\right) t^5 + \frac{1}{720}\left(-a^7 - 3 a^6 b + 103 a^5 b^2 + 509 a^4 b^3 + 509 a^3 b^4 + 103 a^2 b^5 - 3 a b^6 - b^7\right) t^6 + \frac{(a+b)^2 \left(a^6 + 6 a^5 b + 239 a^4 b^2 + 1700 a^3 b^3 + 239 a^2 b^4 + 6 a b^5 + b^6\right) t^7}{5040} + \frac{1}{40320}\left(-a^9 - 5 a^8 b + 472 a^7 b^2 + 8096 a^6 b^3 + 26254 a^5 b^4 + 26254 a^4 b^5 + 8096 a^3 b^6 + 472 a^2 b^7 - 5 a b^8 - b^9\right) t^8 + \frac{1}{362880}(a+b)^2 \left(a^8 + 8 a^7 b + 1012 a^6 b^2 + 26264 a^5 b^3 + 102358 a^4 b^4 + 26264 a^3 b^5 + 1012 a^2 b^6 + 8 a b^7 + b^8\right) t^9 + \frac{1}{3628800}\left(-a^{11} - 7 a^{10} b + 1989 a^9 b^2 + 93619 a^8 b^3 + 834334 a^7 b^4 + 2207602 a^6 b^5 + 2207602 a^5 b^6 + 834334 a^4 b^7 + 93619 a^3 b^8 + 1989 a^2 b^9 - 7 a b^{10} - b^{11}\right) t^{10} + \frac{1}{39916800}(a+b)^2 \left(a^{10} + 10 a^9 b + 4093 a^8 b^2 + 293160 a^7 b^3 + 3555586 a^6 b^4 + 9551772 a^5 b^5 + 3555586 a^4 b^6 + 293160 a^3 b^7 + 4093 a^2 b^8 + 10 a b^9 + b^{10}\right) t^{11} + O[t]^{12}$$

In[52]:=
```
DSolve[
 {
  D[g[t, a, b], t] == a b D[g[t, a, b], a] + a b D[g[t, a, b], b],
  g[0, a, b] == a
 },
 {g[t, a, b]},
 {t, a, b}
]
```

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. ≫

Out[52]=
```
DSolve[{g^(1,0,0)[t, a, b] == a b g^(0,0,1)[t, a, b] + a b g^(0,1,0)[t, a, b], g[0, a, b] == a},
 {g[t, a, b]}, {t, a, b}]
```

In[55]:=
```
DSolve[
 {
  D[g[t, a, b], t] == b D[g[t, a, b], a] + a D[g[t, a, b], b],
  g[0, a, b] == a
 },
 g[t, a, b],
 {t, a, b}
]
```

Out[55]=
```
DSolve[{g^(1,0,0)[t, a, b] == a g^(0,0,1)[t, a, b] + b g^(0,1,0)[t, a, b], g[0, a, b] == a},
 g[t, a, b], {t, a, b}]
```